Electro-Mechano-Fluidic Modelling of Microsystems using Finite Elements

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***Abstract*— Strong couplings between the mechanical, electric, magnetic, fluidic and thermal fields exist in the behaviour laws describing micro-systems. Also, due to the large surface/volume ratio, surface forces such as the electrostatic force and fluid damping become predominant. This paper presents the modelling and simulation of electrically-actuated microsystems taking the electro-mecano-fluidic coupling into account. A micro-resonator consisting in a cantilever beam suspended over a substrate is modelled, and finite element simulations are validated with experimental measurements.**

I. Introduction

From gas and pressure sensors to micro-pumps and micro-mirrors, micro-electro-mechanical systems (MEMS) are used in a large variety of sensors and actuators for automotive, biomedical, environment and space applications. Current simulation technology only addresses the modelling and simulation of MEMS partially, often without satisfactorily solving the strongly coupled, multiscale and multiphysical problems at hand. The aim of this paper is to present the modelling and simulation of electrically-actuated microsystems using the finite element method, focusing in particular on the strong coupling between the electrostatic actuation and the mechanical response, as well as on the modelling of fluid damping. This multiphysic modelling strategy is applied to the simulation of a micro-resonator consisting of a cantilever beam suspended over a substrate. The obtained numerical results are validated against experimental measurements.

II. Multi-Physical Modelling of MEMS

We use the finite element method (FEM) to model the electro-mechanical interactions and to perform a static and a dynamic analysis taking into account large mesh displacements and fluid damping. The usual method to model the coupling between electric and mechanical fields is to use two different numerical codes and iterate between them, which is time consuming and less accurate when the coupling becomes stronger. Here we propose to compute the electric and the mechanical field and their interaction together in the same formulation.

A. Electro-Mechanical Coupling

A consistent way of deriving a finite element discretisation for the coupled electro-mechanical problem consists in applying the variational principle on the total energy of the coupled problem, which includes the electric and mechanical energies. The expression of the energy density results from thermodynamic considerations. If the unknown variables are the displacement and the electric potential, *Gibb's free energy density* G has to be used:

$$G = \frac{1}{2}\mathbf{S}^T\mathbf{T} - \frac{1}{2}\mathbf{D}^T\mathbf{E}, \tag{1}$$

where $\mathbf{T}$ is the stress tensor, $\mathbf{S}$ the strain tensor, $\mathbf{D}$ the electric displacement tensor and $\mathbf{E}$ the electric field, the internal forces may be obtained using the virtual work principle as presented in [1]. Note that the minus sign between the energies comes from the thermodynamic hypotheses. (The electric energy used in this case is actually the co-energy.)

The total energy of the coupled problem on a volume Ω is:

$$W_{int} = \frac{1}{2}\int_\Omega \mathbf{S}^T\mathbf{T}\,d\Omega - \frac{1}{2}\int_{\Omega(\mathbf{u})} \mathbf{D}^T\mathbf{E}\,d\Omega = W_m - W_e, \tag{2}$$

where W_m is the mechanical energy and W_e the electric energy. The variation of the total energy with respect to the displacement $\mathbf{u}$ and to the electric potential ϕ yields the mechanical internal forces $\mathbf{f}_m$ and the electric equilibrium equation q_e, respectively:

$$\begin{cases} \mathbf{f}_m \cdot \delta\mathbf{u} = \delta_u W_{int} = \delta_u W_m - \delta_u W_e, \\ q_e \quad \delta\phi = \delta_\phi W_{int} = \delta_\phi W_m - \delta_\phi W_e. \end{cases} \tag{3}$$

These equations provide the equilibrium equations with the unknowns $\mathbf{u}$ and ϕ. In (3), $\delta_u W_m$ and $\delta_\phi W_e$ can be treated as in the standard variational calculus for uncoupled electrostatics and mechanics. Further, the mechanical energy is independent from the voltage: $\delta_\phi W_m = 0$. The variation of the electric energy due to the displacement $\mathbf{u}$ is the contribution of the electrostatic forces. After some developments [1], we obtain

$$\delta_u W_e \quad = \tfrac{1}{2}\int_\Omega \mathbf{D}^T\mathbf{F}\,\mathrm{grad}\delta\mathbf{u}\,d\Omega, \tag{4}$$

where $\mathbf{F}$ is a matrix function of the space derivatives of ϕ. This term represents the electrostatic forces on the structure. From (3) and (4), a fully coupled finite element formulation can be built following classical discretisation procedures.

The variation of the mechanical and electrostatic forces with respect to small potential and displacement perturbation is an important characteristic of the coupled system since it allows a better convergence of static nonlinear solvers and a better evaluation of the linear vibrations around equilibrium positions. The tangent stiffness matrix around a position $(\mathbf{u}_0, \phi_0)$ may be obtained by linearisation of the internal forces. The finite element form is

$$\begin{pmatrix} \mathbf{K}_{uu}(\phi) & \mathbf{K}_{u\phi}(\phi) \\ \mathbf{K}_{\phi u}(\phi) & \mathbf{K}_{\phi\phi} \end{pmatrix} \begin{pmatrix} \Delta\mathbf{U} \\ \Delta\mathbf{\Phi} \end{pmatrix} = \begin{pmatrix} \Delta\mathbf{f_m} \\ \Delta\mathbf{Q} \end{pmatrix} \tag{5}$$

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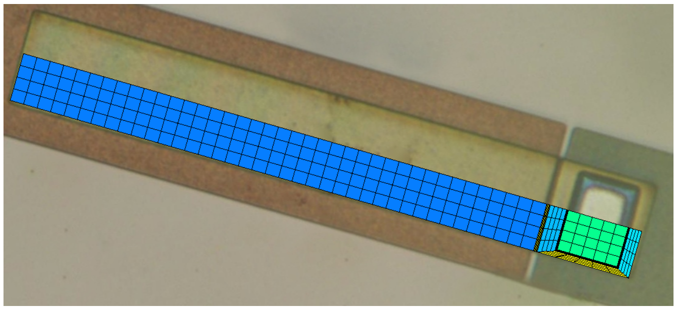

Fig. 1. Finite element mesh superimposed with a microscope photograph of the micro-resonator (viewed from above).

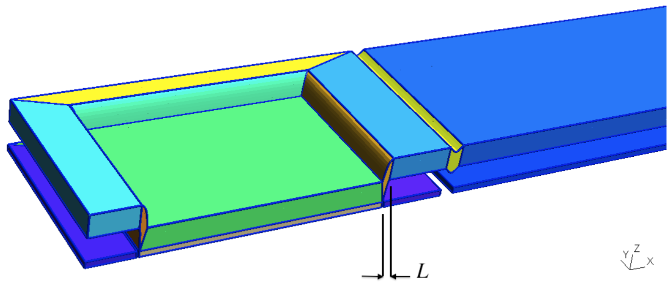

Fig. 2. Detail of the micro-beam anchor.

where $\mathbf{U}$ and $\mathbf{\Phi}$ are the discretised displacement and potential field $\mathbf{u}$ and ϕ. The total coupled matrix is symmetric. The matrix $\mathbf{K}_{\phi\phi}$ is the same as the stiffness matrices of the purely electric problem and need not be further discussed. The other terms are derived from the total energy as presented in [1]. These terms depend on the electric field and the coupled problem is thus nonlinear.

B. Squeeze Film Damping

To model the effect of the air squeezed under the beam, the nonlinear Reynolds equation will be used:

$$\frac{\partial}{\partial x_i}\left(\frac{ph^3}{\mu}\frac{\partial p}{\partial x_i}\right) = 12\frac{\partial(ph)}{\partial t}, \tag{6}$$

where p is the total pressure, h is the distance between the two plates, and μ the air viscosity. This relation is valid only if the flow is laminar and fully developed, if the pressure is constant along the z-direction and the fluid does not not slip at the wall [3]–[5]. In MEMS, as the air layer can become thinner than the mean free path of the air molecules, the viscosity must be further adapted, taking the Knudsen number into account. This will be further developed in the full paper.

III. Simulation and Experimental Measurements

We study a micro-resonator consisting of a cantilever beam suspended over a substrate (the lower electrode). The technology used for fabrication is the PolyMUMPS process proposed by company MEMSCAP. The beam is made of polysilicon and measures $175 \times 2 \times 30\mu m$. The air gap is about $2\mu m$. Fig. 1 shows a top view of the resonator together with the finite element mesh. Fig. 2 shows in more details the geometrical model of the beam anchor.

Fig. 3 compares the static response with experimental data. The strongly coupled electro-mecano-fluidic problem is solved

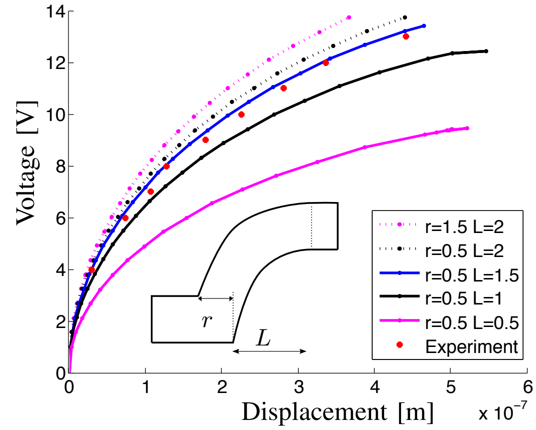

Fig. 3. Applied voltage [V] vs. displacement of the tip of the beam [m] for different values of the length L of the anchor connection (see Fig. 2). The stars denote experimental values.

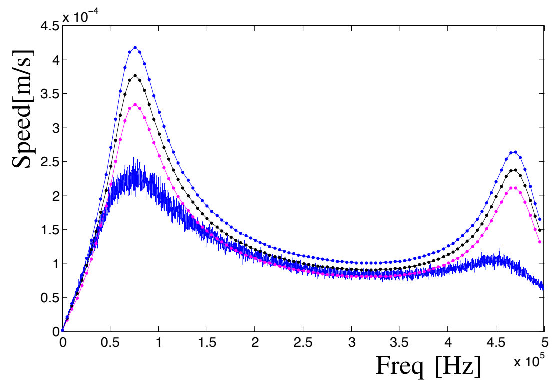

Fig. 4. Dynamic response of the micro-resonator: speed of the tip of the beam [m/s] vs. frequency [Hz]. The jagged curve denotes the experimental values.

using the Riks-Crisfield continuation algorithm [1] to obtain the equilibrium position for for each voltage. The shape of the anchor clearly has a big influence on the equilibrium position.

The dynamic response is presented in Fig. 4. Despite the rather simple fluid damping model, the resonance frequencies measured experimentally using a vibrometer analyser POLYTEC MSA400 [2] match the numerical simulations quite well.

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